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What Is an Economic Shock Wave

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It is now known among governments worldwide, that the LaRouche-Riemann method of economic forecasting has been the only competent forecast for the U.S. economy published during the recent several years. During the same period, since October 1979, other forecasts, including those of the U.S. government, Wharton, Chase Econometrics, and Data Resources, have been consistently wrong to the point of being downright absurd.

The amusing fact is, that in all respects but two, the LaRouche-Riemann method of economic analysis is mathematically the simplest approach to forecasting-analysis in use. Experience has shown that persons trained in physical science and engineering can grasp most of the essential principles of the computer-applications "model" quite rapidly. It has also been demonstrated that an intelligent layman can master most of the principles through study equivalent to a one-semester university course.

The two included features which may cause difficulty even among trained physicists are, first, the rigorous definition of the mathematical representation of "potential" employed for analytical forecasting, and, second, a widespread mystification, even among many physicists, of the notion of hydrodynamic shock-waves, the second of the ostensibly sophisticated features of the model.

In fact, both of these two, ostensibly sophisticated physics-conceptions can be competently described in layman's language. We illustrate that point here. We begin with a broad description of the nature of "shock-waves." Then, we proceed to outline the ABCs of potential theory. Finally, we integrate the two notions, identifying the kind of role the combined notions perform in the LaRouche-Riemann method of economic forecasting.

The ABCs of Shock-Waves

My friends and collaborators, Dr. Jonathan Tennenbaum and Ralf Schauerhammer, enlisted the craftsmanship of a friend to construct a simple, plastic, geometric model of hydrodynamic shock-wave generation. This was originally scheduled to be presented to me on my recent 60th birthday, and was presented only a bit later, to my great delight (**Figure 1**). I requested that Tennenbaum and Schauerhammer present this to the recent conference of the International Caucus of Labor Committees, in aid of my determination to demystify the great Professor Bernhard Riemann's 1859 paper, "On the Propagation of Plane Waves of Finite Amplitude," the paper which is the crucial mathematical feature of the LaRouche-Riemann forecasting method.

Take as an example the simplest kind of hydrodynamic wave, a sine-wave. This can be constructed most usefully by drawing a logarithmic spiral on the exterior of a transparent (e.g., plastic) cylinder, and viewing the resulting construction from the side-view (**Figure 2**).

In schools, it were better that teachers demystify such matters by constructing spirals on transparent cones and cylinders, and that students learn to think about plane waves and plane-surface spirals in terms of plane projections of solid constructions of this sort. By aid of such approaches, based on Jacob Steiner's program for teaching of synthetic geometry, we avoid that mind-deadening mystification of complex functions, which occurs when such functions are presented from a Cartesian or non-geometric, algebraic standpoint of pedagogy.

For purposes of describing shock-wave functions geometrically, we narrow our focus to a half-cycle wave, as Tennenbaum *et al.* have done with the plastic model of the Riemann function (**Figure 1**).

To the best of our knowledge, Leonardo da Vinci was the first to discover and prove experimentally the fundamental principle of hydrodynamics involved. In first approximation, in studying wave-motion in hydrodynamics, we begin with the case in which a wave, such as this sine-wave, moves across the surface of the water, but without moving water in the direction of lateral motion of the plane wave. In other words, as the wave passes along the surface of the water laterally, it moves the water up and down, but not forward. *In other words, we recognize the existence of conditions such, that a wave moves hydrodynamically in the universe without moving matter in the direction of lateral movement of the wave.*

Recollect standing at the edge of the sea, watching waves moving toward shore, forming breakers as the shore is approached (for example). In the case that there is motion in the same direction as lateral movement of the wave across the surface of the water (for example), exactly how does this sideways movement within the wave occur? The plastic model constructed by Tennenbaum *et al.* shows in principle how this occurs.

Let us call this lateral movement, movement "toward the shore." In what part of the wave does the greatest relative movement toward the shore occur? In brief, at the base of the wave, the change of relative movement toward the shore within the wave approximates zero, whereas the maximum increase of movement toward the shore occurs at the peak of the wave. The rate of relative movement toward the shore increases from zero to the maximum rate as we trace our eye's movement upward toward the peak (**Figure 3**).

See this same argument in terms of the plastic model (**Figure 1**).

In explaining this to schoolchildren, or others beginning their acquaintance with such matters, we illustrate our general thought by aid of approximate truths. We say that our introduction of the idea of hydrodynamics begins with observations and experiments using an approximately incompressible fluid, water. Once we have mastered some basic features of hydrodynamic behavior of water, we look around us, to discover cases in which other media behave according to hydrodynamic principles.

In explaining sonic booms to children, for example, we point out that as an object moves through the atmosphere near the speed of sound, the air becomes very much like water in one respect: it becomes a relatively incompressible medium, relative to the movement of the body. We say, that as a result, the air behaves, in some significant respects, as a hydrodynamic medium, generating the shock-wave we identify as the sonic boom caused by a supersonic aircraft's flight or a supersonic bullet's trajectory.

So far, the whole matter might appear quite straightforward. Therefore, why should there have been any controversy among physicists concerning the conclusions projected by Riemann's 1859 paper, in which the generation of such "sonic booms" was first analyzed and predicted?

During the 1890s, Lord Rayleigh, Bertrand Russell, and others, insisted that Riemann's physics was absurd. Rayleigh, in particular, insisted that "sonic booms" could not exist. The reason for that hullabaloo is, that if Riemann's physics is correct, if sonic booms are generated in such a fashion, then there exists a fundamental absurdity in the kinds of mathematical physics associated traditionally with such figures as Descartes, Newton, Cauchy, Maxwell, Helmholtz, Kelvin, *et al.* The real universe could not be the kind of universe the mathematics of Newton-Cauchy-Maxwell imply.

In other words, the kind of physics Riemann brought to bear upon his 1859 "shock-wave" paper implies a different kind of universe than the Newton-Cauchy-Maxwell school insists to exist. The organization of the universe is not Newtonian, but is, rather, hydrodynamic.

It is my own chief contribution to scientific work to have discovered and demonstrated, beginning 1952, that the ordering of economic processes corresponds uniquely to the implications of Riemannian physics.

The central role of Riemann's 1859 paper in the computer-applications "modeling" for LaRouche-Riemann forecasting is not some clever trick with mathematical analogies. Economic processes are characterized by shock-wave-like transformations, because economic processes are hydrodynamic in their most characteristic features. For appropriate reasons, I have stipulated that an economic process must be thought of by physicists (for example) as a thermohydrodynamic process.

The problem is best understood by a thumbnail outline of the historical background to the Riemann-Maxwell controversy.

Modern Science

Modem science begins with the commentaries on the work of Archimedes by the 15th century's Cardinal Nicholas of Cusa. The explicit development of modern science began in Milan, Italy, through the collaboration of Leonardo da Vinci and Luca Pacioli, in the course of which Leonardo assimilated and richly elaborated Cusa's discoveries respecting scientific method.

This work led into the establishment of two interacting schools of French and German science established near the close of the 16th century, typified by Johannes Kepler and Gaspard Desargues, respectively. The work of Kepler and Desargues was brought together chiefly by Gottfried Leibniz, during the 1671–1676 period of Leibniz's initial completion of development of his differential calculus (submitting the discovery of the differential calculus to a Paris printer in 1676). So, we have Kepler leading into Leibniz on the one side, and Desargues, Fermat, and Pascal leading into the work of Huygens and Leibniz on the other side. The effort is brought together under the patronage of France's Jean-Baptiste Colbert.

This work was continued through the 18th century by Leibniz's followers in Germany, Switzerland, Sweden, and Russia, and by the Oratorian teaching-order in France and Italy. Over the period 1794–1815, the center of scientific and technological progress internationally was the French Ecole Polytechnique under Gaspard Monge and Lazare Carnot, both products of the Oratorian teaching-program.

It is a simple matter of historical fact, that every important, fundamental scientific discovery effected into 1815 was accomplished exclusively by the current leading from Cusa into the joint work of Carnot and Leibniz's followers in Germany.

When Laplace and Cauchy combined efforts virtually to outlaw science from France, Alexander von Humboldt and Carnot organized the transfer of French science to Germany, initially chiefly to the University of Berlin and the Prussian Military School, with a simultaneous transfer of French science in large chunks to West Point under Commandant Sylvanus Thayer.

Especially from the early 17th century onward, the teaching and explanation of science has been divided into two irreconcilable factions. The faction typified by Cusa, Leonardo, Kepler, Leibniz, *et al.*, was termed by its British adversaries the school of "continental science." This "continental science" faction was distinguished by its insistence that the laws of the universe were geometric, not algebraic in form. The opposed British, or "reductionist" faction, insisted that the laws of the universe were axiomatically anti-geometric, algebraic in form.

The issue of Kepler's discoveries is at the center of this controversy (not the fraudulent issue of "Copernican man" occupying the arguments of such British agents as Arthur Koestler).

"Continental science" is entirely derived directly from development of a conception first known to have been presented in the *Timaeus* dialogue of Plato, the conception associated with the so-called Five Platonic Solids. Riemann's special significance in modern science is that he was the first to complete a successful method for designing comprehensive experiments by means of which the Platonic character of the universe's lawful ordering could be rigorously proven in a general way. The 1859 "shock-wave" dissertation is the most typical of the experiments which Riemann designed to prove this fact.

During Plato's lifetime, one of his collaborators, working at the Cyrenaic temple of Ammon, was the first known person to have proven that there are only five types of regular, polyhedral solids which can be constructed in Euclidean space, a proof most rigorously reconstructed by Leonhard Euler during the 18th century, and also reconstructed by Luca Pacioli earlier.

From this demonstrated fact, Plato adduced several interrelated conclusions which are central to the *Timaeus*. These conclusions were later proven for the ordering of the solar orbits by Johannes Kepler, proving an hypothesis earlier developed by Leonardo da Vinci *et al.* The followers of Kepler and Desargues developed Kepler's discovery, in a pathway of development leading chiefly through the Ecole Polytechnique, and through Humboldt and Göttingen universities during the middle 1850s and 1860s.

The fact that only five Platonic solids could be constructed in visible space proves that visible (Euclidean) space is bounded by limiting geometrical principles. Plato argued, visible space is not a direct representation of the real universe, but is rather a lawful reflection of the real universe, a reflection seen in "distorted" form in a mirror, a mirror everywhere embedded in the real universe. Plato also insisted that the distribution of events in the mirror is governed by harmonic principles, and that we must master those harmonic principles in order to adduce the real, unseen universe reflected to us as visible space.

Using the discoveries of Leonardo *et al.*, Kepler designed an experiment, to test whether or not the ordering of the solar orbits was fully consistent with such harmonic distributions of events. With aid of certain corrections, made possible through development of complex functions, we must say that Kepler's laws uniquely, exclusively account for the fundamental principles of astrophysical phenomena today—whereas the Newton-Cauchy-Maxwell program does not.

The conclusiveness of Kepler's proof was finally confirmed by young Carl Gauss. Kepler specified that if his laws were uniquely correct, and all alternative assumptions necessarily wrong, then there must have existed once an exploded planet in an orbit whose harmonic orbital values he specified. It was later discovered, first by Carl Gauss, that the asteroid belt had precisely the harmonic orbital values prescribed by Kepler.

The sum of the mathematical work flowing from Kepler's discoveries was brought to an intermediate conclusion chiefly by one of the leading figures of the Ecole Polytechnique, Louis Lagrange. Riemann worked to complete the work of Lagrange, aided chiefly by the crucial discoveries of one of Riemann's immediate teachers, Lejeune Dirichlet.

What Riemann accomplished is summed up in preliminary form in his 1854 "On the Hypotheses Which Underlie Geometry." The kernel of that dissertation was a comprehensive, if preliminary set of general specifications for design of what Riemann described as "unique experiments," of which the 1859 "shock-wave" paper is a notable illustration.

The central question of scientific inquiry is, given the fact that the real universe is unseen by us, under what conditions and by what methods can we adduce valid laws for the unseen universe from experimental observations made in terms of reference to phenomena of visible space? Riemann defined visible space, the distorted reflection of the real universe seen by us, as a *discrete manifold*, and the real universe reflected into the mirror as the *continuous manifold*. Under what special experimental conditions can we be assured that certain selected statements about observed relations in the discrete manifold are also true for the continuous manifold?

To summarize as much of the matter as is directly relevant to our discussion here, the kinds of experiments through which we may develop valid statements about the universe, which Riemann named "unique experiments," involve qualitative changes in the lawful ordering of processes observed in the discrete manifold, changes of the sort we often associate with the name "relativistic phenomena."

In other words, to the extent experimental observation focuses only upon the kinds of mathematical formulae which simply repeat themselves over and over, we are able to construct statistically "provable" mathematical descriptions of nature which either may or may not actually correspond to the lawful ordering of the universe. It is only as we conduct experiments in which we appear to change the local laws of the universe, that we are discovering the lawful principles delimiting the kinds of such change the universe permits.

The convenient name for the kind of experimental inquiry which focuses directly on those special, unique kinds of cases, is "relativistic physics." *Only experiments which are immediately focused on relativistic phase-changes in observed processes tell us valid things about the lawful ordering of the universe.*

The paradigm for the anti-scientific view of the universe is not Newton, but Descartes, Newton is merely a degenerate version of Cartesian arguments. Descartes' universe is a "big bang" universe. "God created the universe one day, and thereafter became impotent to change the composition of laws he had created."

For Descartes, the real universe is nothing more nor less than the discrete manifold—empty space, stretched infinitely, in which particles move about, acting upon one another. It was this particular absurdity, this dangerous absurdity, in the scheme of Descartes which was ruthlessly attacked by Pascal, Leibniz, and by the Ecole Polytechnique. Although the Cartesian scheme may appear to some to be geometric in. conception, it is really a naive Euclideanism which leads directly to an axiomatically algebraic, or, to use a more ancient name, cabalistic, conception of mathematical physics.

Algebra is essential a psychologically poisonous Phoenician (Philistine) cult superimposed upon the body of scientific work.

The original curriculum of the Ecole Polytechnique was entirely, pervasively geometric, a feature of the program directed by the great geometer Gaspard Monge. In 1816, when the House of Orléans had subjugated France, Laplace took over direction of the Ecole from the exiled Lazare Carnot. Laplace ripped out the geometrical curriculum from the Ecole's programs, and superimposed his own cabalistic, algebraic scheme. Laplace's fanaticism was complemented by the hoaxster and plagiarist Augustin Cauchy. Their combined efforts created the conditions of inquisition against French science which compelled French science to exile itself in Alexander von Humboldt's Prussia.

Although Professor Felix Klein and his collaborators defended most of the fundamental accomplishments of their French and German forebears, over the past hundred years, the influence and knowledge of the method of "continental science" has been ripped out of university curricula, and reduced to knowledge of a now-near-vanishing portion of the science profession.

In Germany, which was the paradigm for fundamental progress in science into the early 1920s, science was already being systematically destroyed under Weimar. With Hitler's accession to power, Nazi hacks rapidly displaced competent scientists in most key university positions, with only a fragment of competent German science surviving during the Hitler-period. This temporarily revitalized science in the U.S.A., but we have refused to reproduce the reinfusion of scientific excellence these émigré-scientists brought with them.

In Germany today, it is chiefly only a dwindling number of the students of Werner Heisenberg who represent competence in fundamental work. When they retire, scientific competence in Germany will have disappeared.

Today, only by exception do we support important varieties of fundamental research and correlated development in physics and related fields, and that chiefly through government-backed science-driver projects such as the pre-1967 NASA effort. Beginning with the introduction of the "New Math," we have permitted the destruction of competent pre-science teaching in public schools, and have adjusted university programs to the predominant incompetence public-school graduates have cultivated.

With a diminishing percentile of exceptions, public-school graduates from the class of 1966 onward are less rational, less able to assimilate technological skills, as well as scientific competence, than the classes of the earlier period. Competent teaching of geometry—the foundation of competence in scientific thinking, or skilled use of machine-tools—is vanishing from education.

Fundamental conceptions which might have been rather readily assimilated by earlier generations of secondary-school graduates, seem both very mysterious, and even infuriatingly wrong, to most of the past 15 years of secondary-school graduates. Even among most of those who are broadly qualified professionals, the implications of thermohydrodynamics, as they apply, in particular, to economic processes, appear as either very mystifying or even flatly wrong.

So, the Riemannian conception of a potential function, or the use of the shock-wave function in economic analysis, must tend to appear a very strange business to most today, where the same point would have been grasped more readily by professionals even less than fifty years ago.

All crucial transformations occurring in the real universe have the mathematical-geometric form otherwise exhibited in shock-wave generation. In each instance the nature of the process being analyzed requires us to recognize such a crucial (e.g., relativistic) kind of transformation as occurring, our analytical task is to apply a mathematical procedure analogous to the shock-wave function, and to determine experimentally the boundary-conditions, and phase-space parameters corresponding to the expression of that function in that case.

Hence, "LaRouche-Riemann method."

Potential Function

The existence of mankind can be measured functionally in only one way, as a process of increasing (or decreasing) relative population-density. How many persons per square mile, can be sustained by the labor of society at existing levels of technology of practice?

Our objective is not merely to produce an increase in raw population. Our objective is to produce a population capable of increasing further the relative population-density. What we must desire to produce is an enlarged population with an increase in per-capita potential relative population-density.

Restated in mathematical-like terms, we are obliged to accept something which is most unsatisfactory from the vantage-point of an axiomatically algebraic world-outlook. Our rigorous statement of the principle involved requires a mathematical notion of the sort some have termed "self-reflexive functions." The radical cabalist-cultist Bertrand Russell and his Cambridge University friends have had public rug-chewing fits over the suggestion that self-reflexive functions exist.

In other words, if we examine human activity by the standard of perpetuation of human existence as a whole, the significant feature of per-capita human activity today is its realized potential to produce an enlarged population with an increased such potential tomorrow.

Unfortunately, the usual procedure in economics practice today, is to interpret economic performance from the standpoint of the ordinary bookkeeper or accountant. Economic performance is measured either as the number of things produced, or, worse, as the aggregate net price of total purchases and sales occurring in an economy. Public opinion is so thoroughly conditioned to delude itself that accountants are economists, that mere statistical accounting of things or bookkeeping values is credulously swallowed as economics practice. Such delusions made possible the career of the dangerously incompetent Robert Strange McNamara.

It is true that production of useful things is the visible form of the activity on which human existence depends. It is an easy matter to prove that things have no economic value in and of themselves. The question posed to economic science is: What value do particular things produced have for the perpetuation of society's existence? *Does this production of things increase or decrease the per-capita potential relative population-density of the new generation?*

Let P signify a general term for per-capita potential relative population-density. P_1, P_2, P_3,..., P_i,..., P_m,..., then describes the rising per-capita potential relative population-density over successive periods of development of an increasing total human population. This series implies a corresponding function, $F(P)$, of some kind. The value associated with this function either increases (normatively) continuously over time, or the society becomes sick or even begins to die.

The significance of things produced is the effect of their production and consumption on increasing the value assigned to that potential-function.

That is not yet rigorous enough. In place of a series of successively rising values, we require a special variety of fixed value for our function, a fixed base-line value for the function as a whole statement. This fixed value for the function as a whole subsumes epoch-to-epoch increases in per-capita potential relative population-density. This value for the function as a whole has the significance of what we might sometimes wish to name "a world-line." This value implies the pathway of self-development of society which is the minimal rate of improvement of per-capita potential relative population-density required to sustain human existence indefinitely.

Consequently, in economic science properly elaborated, we define potential as the power to produce increased potential of the same kind.

I appreciate the initial difficulties experienced even among professionally trained persons. I wrestled long with this sort of notion from my first attempt to master some of Leibniz's work, at the age of 12, and did not begin to reach a satisfactory overview of the matter until the age of 30, after nearly a year of grappling with the implications of Georg Cantor's notion of the transfinite. Without recognizing, finally, the significance of Cantor's work, I would not have grasped independently the significance of Riemann's methodological approach. Thirty years after that, I may hope I have become sufficiently a master of this conception that I might put myself forward to make it accessible to others more generally, to reduce the issue involved to experimentally demonstrable terms of reference without compromising anything fundamental.

I indicate the significance of "transfinite," and then restate the point which I have just cited.

The bare-bones idea of a "transfinite" magnitude may be developed as follows.

The simplest approximation of a transfinite number is the counting of integers. In first approximation, this may be stated that for the case that n identifies the largest integer we have counted by this method so far, the next integer is identified as $n+1$, or $n-1$. By stating as an idea the procedure of counting by which all members of a class may be counted, one introduces the substituting of the state of such an idea for the detailed counting of each and every number of the class.

For example, we ask ourselves how we count in a rigorously orderly fashion all of the fractional numbers (including integers), otherwise called rational numbers? By thinking about the construction of the Sieve of Eratosthenes, we have reduced all rational numbers to a single idea. We may do the same for the class of irrational numbers (e.g., roots which are not otherwise rational numbers). We may do the same for the class of algebraic numbers. And, so on and so forth.

The generalization of such ideas in modem science begins, as a significant development, with the work of Fermat and Pascal during the middle of the 17th century. Pascal elaborated the principle that meaningful series of ordinary numbers are defined from the standpoint of geometry, not arithmetic. This established the arithmetic features of the differential calculus, which Leibniz combined with Kepler's specifications for such a calculus, to develop the first generalized form of the modern differential calculus by 1676.

Leibniz's treatment led into Leonhard Euler's related work in defining the basic formulations for topology, and related matters.

The notion that arithmetic series, and algebraic functions are merely by-products of purely-geometric functions is an ancient idea of uncertain age, and is the kernel of competent varieties of modern treatment of algebraic functions. The development of the notion of the transfinite by Cantor during the period 1871–1883, is essentially a continuation of, and generalization of that point.

Since all arithmetic and algebraic functions can be reduced implicitly to a statement in geometry, we are able to "handle" large, unlimited arrays of numbers, etc., as in a single act of thought, by discovering the appropriate geometrical idea which generates the array as a well-defined collection.

Among the most interesting of such ideas, as it occurs within the limited bounds of arithmetic as such, is the determination of the number of primes in any counting-interval between 0 and some chosen integer *n*. Fermat reported he had discovered a solution for this —and might well have been correct in that report. Leonhard Euler tackled the problem afresh during the 18th century, and Riemann reworked Euler's conception approximately a century later. Analytically, this discovery of the Riemann-Euler function for primes has never been analytically disproven, though a full appreciation is still wanting. The idea of substituting the Riemann-Euler function for an actual counting-out of those prime numbers within the interval, belongs to the general kind of thinking-behavior associated with the notion of transfinites.

This Riemann-Euler function is most fascinating not merely because of the popularity of the determination of prime numbers. It says implicitly that the only real numbers in the universe are what we term complex numbers, or, rather, complex functions: e.g., transcendental numbers, and that the integers, and all so-called "real numbers," are merely singularities within the continuous domain identified with complex functions.

From the standpoint of Riemann's work (most emphatically), such statements have a precise and fundamental significance. The point is illustrated by our summary description of wave-functions earlier. We derived a sine-wave function on a plane from a spiral on the cylinder enclosing that plane, showing how the number π and the natural-logarithmic base e enter into the trigonometric function by way of simple geometric construction.

Look now at **Figure 4**, Self-Similar "Growth" Spirals. We shall refer again to this figure at a later point in our presentation. This is a conception we owe originally (to the best of our present knowledge), to Archimedes, and to later work of Luca Pacioli and Leonardo da Vinci.

We used the hypotenuse of right-triangle A as the long leg of the similar right-triangle B, and used the same procedure to develop successively C, D, E, F, G, and H. The result approximates the growth of a snail's, conch's or nautilus's shell, approximately an Archimedean spiral.

Two things must be said about this immediately. First, the Archimedean spiral is a spiral whose relative proportions are those of the Golden Section (sometimes misleadingly named the Golden Mean). This Golden Section arises in the construction of the Five Platonic Solids in constructing the pentagon, and corresponds to the crucial harmonic proportion of the fifth in music and in astrophysics (as elsewhere). This ratio is characteristic of two things in our universe's discrete manifold: the morphology of living processes and the universe's ordering as a whole.

It is approximated in arithmetic by what is called Fibonacci's series, the hypothetical calculation of the growth of populations of rabbits. As the Fibonacci series becomes relatively large in numerical magnitude, that series converges upon the Golden-Section determined geometric growth-pattern.

This spiral is properly generated in synthetic geometry by constructing a self-similar logarithmic spiral on a cone. The image of the spiral projected from the side to the base of the cone is the Archimedean spiral. Construct a hexagon inscribed in the circle of the cone's base. Derive a regular twelve-sided polygon, inscribed in the circle, from that hexagon. Divide the circular base of the cone, in this manner, into twelve equal, circular sectors. The radii will then intersect the Archimedean spiral in precisely the proportions of the length of the spiral corresponding to the well-tempered scale. This was presented by Dr. Tennenbaum in 1981.

Now, proceed to the next step in Dr. Tennenbaum's constructions of the well-tempered scale. Using a procedure like Euler's inscription of the Five Platonic Solids within a single sphere: The Keplerian intervals—inscribed within a cylinder, from point to point of a logarithmic spiral—yield the proper musical progressions.

Now, go a step further. Treat the cylinder as a very elongated cone, and project the result upon the cone's base. This portrays the significance of differences of musical register as well as the self-similar proportions of the well-tempered scale throughout the span of registers.

Let the result of any such projection be treated as a reflection of the continuous manifold projected into the discrete manifold of Euclidean space. The geometrical notion of the generative continuous manifold's images, accounts for the projected images as a class.

Such pedagogical ruses demystify complex functions in a preliminary fashion, helping the student to think rationally about the connections between continuous and discrete manifolds. This kind of thinking underlies the deeper significance of the notion of transfinites.

Generally, transfinite "numbers" are geometrical conceptions which compact thinking about a complex extension of a definite class of determined particular things into a single action of thought. It is thinking directly about the universals which lawfully determine large arrays of particular things. Ontologically, it is the way in which we can recognize, in a rational way, the reality that reality is located with universals, and that particular things (singularities) are relatively ephemerals determined by such universals' self-elaboration.

This approach enables us to proceed to study of the interaction among universals as universals. Our function, F(*P*), is such a transfinite.

It is the popular persuasion, that grasp of fundamental principles of scientific work depends upon working one's way through years of apprenticeship, successively mastering ever-more-complicated constructions in mathematics. Perhaps, after 30 years of graduate studies and assistant-professorship's research-activities, one's head might be sufficiently stuffed with refined knowledge that one might be able to begin to attack fundamental questions. I exaggerate to make the point.

On the contrary, the really fundamental questions of scientific method are those typified by Plato's appreciation of the implications of the Five Platonic Solids. Most of the important errors in scientific work are not the sort of errors one associates with correcting an algebraic formulation (or, some spy's stealing a "secret formula"). All of the important errors in scientific work are elementary errors. The important errors are those assimilated, or left uncorrected at the age of 6 to 16. These errors of assumption become embedded, as by an "hereditary principle," in the elaboration of mathematical and other constructions all the way to the status of professor emeritus. Truly accomplished professors emeritus are of the sort who recognize that a major problem of science today might be the ingenuous acceptance of a wild error asserted by Michael Faraday, for example.

This is the history of scientific progress, in which all truly fundamental achievements were exactly a rigorous criticism of one or more of the most commonplace, "elementary" assumptions which, proverbially, every educated professional passively accepted as true.

The enemy of scientific progress is the popular myth that something profound must necessarily be very complicated, as requiring blackboards strewn with densely-packed analytical treatments of algebraic functions. Contrary to this, insight proximate to fundamental discovery is illustrated by the insolent fellow who goes to the messy blackboard, erases some of the chalky fustian thereon, to make a single, simple geometric diagram, which goes directly to the crucial, elementary issue of the entire matter.

For example, to the best of my knowledge, Cusa was the first to discover the fundamental principle of rigorous topology: that the circle is the only self-evident figure in geometry, and that points and lines have no axiomatic existence. One constructs a "straight line" by folding a circle against itself—any other definition of "straight line" leads to wild absurdities in the physics of multiply-connected manifolds. Similarly, one constructs a point by folding a semi-circle against itself. Lines and points are determined (constructable) singularities of circles. Once that correction to popular mythology is made, mountains of elaborate algebraic rubbish fall more or less immediately from the corpus of mathematical physics.

"But," the attempted rebuttal is heard, "we construct a circle by rotating a line around a point." An understandable blunder is embedded in that argument, a blunder whose character is a lack of sufficiently rigorous attention to fundamentals. The significance of the circle is that it represents, first, closure in the discrete manifold. In elementary topology, the student is introduced to the proof that the circle is the closed curve which circumscribes the relatively largest area. That is necessary pedagogy, but is not yet at the bottom of the problem. What is the physical significance of enclosing the relatively largest area? What are the crucial implications of such a statement? It is the student whose mind is troubled by scent of such implications who will probably become the great scientific discoverer of tomorrow. "True, we construct circles with a compass, but what is it we have constructed? What previously existing form of existence in the universe have we copied? What is the nature of our universe, that circles might be constructed in such a fashion?"

This sort of rigorous scientific thinking was associated with 15th-century and later reexamination of the implications of the Five Platonic Solids. By choosing, primarily, the regular polygons corresponding to the facets of the Five Platonic Solids, and by inscribing those in a circle, Kepler reproduced Plato's notion of harmonic intervals. This has been, in historical fact, the procedure upon which all competent varieties of modern mathematical science have been constructed. Is one's mind not properly fascinated to understand why that is the case, how the universe is constructed to the effect that this has been the case? All rigorous mathematics begins with the topological principle, that the circle is the only self-evident geometric form existing in the discrete manifold. Through the initial singularities, the "point" and "straight line," derived by construction from the circle, every possible geometric form existing in the discrete manifold is rigorously derived, as a product of the circle. That is the first mathematical law governing all phenomena of the discrete manifold. Therefore, the essential, elementary feature of all competent varieties of mathematical proof is a synthetic-geometrical demonstration that the specific geometrical form examined is coherent with its lawful derivation from the circle or sphere. Consequently, no mathematical formulation is acceptable in competent mathematical-physics discipline, unless the proponent of the formulation first constructs the geometric model which an algebraic formulation purports to describe. If a mathematician can not indicate the geometric model, then we must say of his algebraic formulation, that he does not yet understand what he is talking about. We examine the geometric model of the function according to principles adduced from the principle of synthetic geometry, that all geometrical existences in a discrete manifold must be proven with respect to circular or spherical derivation: closure.

Most of the formal mathematical fallacies encountered in scientific work can be reduced to the matter of failure to adhere to the rigors of geometrical closure.

The same state of mind, the same focus on rigorous examination of elementary notions, is central to the notion of per-capita potential relative population-density in the LaRouche-Riemann method. From the outside, the argument employed to develop the formulations for per-capita potential relative population-density is easy to follow. Yet, this easy-to-follow argument leads us to clear and readily understood conclusions which are violently contradictory not only to differing approaches to political-economy, but sharply offensive to numerous among physics and mathematics professionals. Wherever such an experience arises in scientific work, we know that there exists some extremely clever and resourceful fallacy embedded in the presentation of simple, elementary conceptions. The fallacy of this elementary nature exists either in the offending doctrine being offered, or in the contrary doctrines which the presented material sharply offends. It is so here. The conception we employ is clear and simple to follow, elementary, and yet in that elementarity lurks one of the most profound issues in scientific work in general.

At this moment, we turn to focus upon the embedded, elementary conception underlying the LaRouche-Riemann method of defining per-capita potential relative population-density. In this way, we bring forward the key issue of method which the mathematical-physics professional must suspect to exist. Once this issue is clear, the implications of the LaRouche-Riemann notion are no longer obscure.

Science's Roots in Judeo-Christian Neoplatonism

Since the work of Philo of Alexandria and the early Christian Apostles, "Neoplatonism" has meant, in its proper usage, a kind of superimposition of Judeo-Christian principles upon the methodological world-outlook reflected most emphatically in the *Timaeus* dialogue of Plato. This superimposition is not of the form of an encasing of Platonism within a Judeo-Christian theological confinement. Rather, Judeo-Christian Neoplatonism goes directly to the heart of Plato's conception, and makes an explicit statement of policy respecting that conception.

All fundamental achievements in modem science have been adduced directly from that root. So, it is not properly astonishing that the greatest theologian and law-giver of the modem era, Cardinal Nicholas of Cusa, should have been in essence also the founder of modern science.

In Philo's exposition of Judaism, and in the work of Christian Apostles and leading patristics, beginning with the opening passages of the Gospel of St. John, Neoplatonism has these chief, summary distinctions.

First, Christianity (in particular) attacks directly and without compromise or toleration, all pagan cults of the Hesiodic, Phoenician, and other "Great Mother" varieties. "Great Mother," whether in the guise of Cybele, Mithra cultism, Sakti, Isis, is for Christianity "the Whore of Babylon." The worship of Lucifer-Apollo-Horus, etc., and of Satan-Osiris-Dionysos-Siva, is correctly appreciated as a subfeature of the worship of the "Great Mother," Isis or the "Whore of Babylon."

Second, Judeo-Christian Neoplatonism rejects all population-equilibrium doctrines, including those attributed to ancient Platonism. This is chiefly identified with the injunction of the Book of Genesis, that mankind must "Be fruitful and multiply, and fill the earth and subdue it." This injunction translates directly and necessarily into the form of per-capita potential relative population-density.

Third, the power of mankind to perfect its mastery of nature, as through scientific-technological advances in the productive powers of labor, is implicit in the creation *Filioque* doctrine of the Latin liturgy. This goes beyond the Composer-Logos consubstantiality of the *Timaeus*, although agreeing fully with Plato's notion of the principle of consubstantiality. In Christianity, the creation of Christ to be a consubstantial part of God the Composer is God's enlargement of Himself. Man, through his embedded divine potential, must imitate Christ, to become an instrument of the process of continuing creation, to develop further the universe in a creative manner, to the Glory of God.

Fourthly, Judeo-Christian principles reject absolutely the "big bang" version of one-time creation typified by the doctrine of Aristotle. Philo, for example, is explicit on this point. The universe is an unfolding composition, a process of continuing creation. The lawfulness of the universe's composition is not confined to fixed laws of a mechanical (e.g., Cartesian or Newtonian) variety. The lawful principles of the universe are rules governing the unfolding of continuing creation, a process of continuing creation in which fixed, mechanical-type laws are successively transformed.

This has been the standpoint, inclusively, of the fundamental contributions to science by Cusa, da Vinci, Kepler, Leibniz, and Riemann. Riemann, for example, is most explicit in this, as in his treatment of Herbartian antinomies. Friedrich Schiller was also directly explicit on this point, in his criticism of the fundamental methodological fallacy in the work of Immanuel Kant.

Man must fulfill the injunction to exert increasing dominion over the universe, by discovering more perfectly those higher laws of continuing creation (as distinct from ephemeral, mechanical kinds of consistency in a discrete manifold). This perfected adducing of the lawful ordering of continuing creation is the proper content of science.

Although this is the explicit content of Judeo-Christian Neoplatonism, it is an outlook not unique to such Neoplatonism. The establishment of Israel and the reestablishment of Athens occurred not accidentally within approximately ten years of one another. In the case of the refounding of Athens, it was the sponsorship by the temple of Ammon which was critical, and there are strong internal indications that this affects the case of Moses, who led the chosen group who became the Israelites, to overcome the power of the Philistines. (Knowledge of the evil represented by Cadmus, and by Thebes, informs our view of the ancient Philistines in this matter.) The view of God, man, and nature associated with the adversaries of Hesiodic dogma in Greece—Solon, Aeschylus, Plato, *et al.*—is known to have developed under the patronage of the temple of Ammon, in opposition to the Isis-cult within Egypt.

It is a matter of work in progress, that this writer and his collaborators are working with Brahmin and related scholars to sort out a like current within Vedic philosophy-theology, in which the principle of *life-continuing creation* represents the leading positive current. Correlated with this aspect of that inquiry is a study of the astronomy reflected in Vedic and related sources, as earlier examined by Kepler and as studied in German circles including Carl Gauss and August Böckh. This latter matter is referenced substantially in the writer's recent book, *The Toynbee Factor in British Grand Strategy*. Dr. Uwe v. Parpart has provided an appendix for that book, in which exemplary cases of ancient astronomical cycles-knowledge are provided for the reader's reference.

Contrary to British doctrine on the ice-age phenomenon, scholarly research in Germany has shown that the entry of the Gulf Stream into the Arctic region during the second part of the past million years must have shifted the ice cap from the Arctic to the adjoining continents, to effects coinciding with the findings of Bal Gangadhar Tilak. The Arctic astronomy reflected in the ancient Vedas, including an immensely great long-wave cycle verified first by Kepler, and accurate cycles for migration of both the geographic and magnetic north poles, is conclusive evidence bearing on the theses included in both Tilak's work and in the recent *The Toynbee Factor*.

Long before the evil cult of astrology appeared, there existed a rigorous, empirically grounded science of astronomy-navigation. Science and the forerunners of Judeo-Christian Neoplatonism emerged in tandem, as man lifted his eyes from the bestiality of groveling in the moral muck of blood and soil, and turned his eyes to the heavens, to discover his proper and meaningful place in the unfolding of continuing creation.

So, although our primary reference here is Judeo-Christian Neoplatonism, our view of the matter is also much broader.

This is key to Georg Cantor's notion of the transfinite, the standpoint from which Riemann's work is properly to be assessed. Cantor's notion was not essentially original to him. Rather, the work of both Riemann and of Cantor's immediate predecessor, Karl Weierstrass, afforded the most advanced view of the combined work of two leading figures of Carnot's Ecole Polytechnique, Fourier and Lagrange. This advancement enabled Cantor to reformulate the pre-existing notion of transfiniteness on a new mathematical-geometric basis.

By transfinite, we mean that reality, substance, ontology, exists only in the continuous manifold, rather than in the reflected images of the discrete manifold. Transfinite existence is not a mere construct adduced from algebraic orderings of phenomena of the discrete manifold. Transfiniteness is not merely a superior method of mental construction for conceiving of the ordering within a discrete manifold.

This is illustrated by examining the equivalence of Weierstrass's treatment of ordered discontinuities and Riemann's geometrical treatment of discontinuities in such harmonically ordered forms as shock-wave generation. The continuous manifold is uniquely the ontologically real. It is comprehension of the reasons, the proofs, that this is necessarily the case, which locates the deep roots of the elementary notions of the LaRouche-Riemann method of economic forecasting.

For example, A. Einstein's $E=mc^2$ is not consistently Riemannian relativism, even though Einstein did in fact owe most of his own approach to relativistic physics to the Kepler-Riemann notion of relativism. Between Riemann and Einstein there intervened the corrupting influence of Helmholtz and his circle of de facto Isis-cultists, allied to the Isis-cultism dominating scientific discussions and policy of Britain's Cambridge University during and following the period of Clifford's work. The corrupted notion of "energy" associated with Helmholtz and his circle, plus the pernicious influence of L. Kronecker and the delphic Richard Dedekind, introduced a reductionist, mechanistic misdefinition of "energy," to the effect that the *E* of Einstein's $E=mc^2$ is an intrinsic obstacle to coherent relativism within Einstein's doctrine.

This Helmholtzian notion of "energy" shifts the location of ontological reality from the continuous manifold back to the Cartesian discrete manifold, to the effect of attempting to reverse absolutely everything earlier accomplished in the development of modern science, from Cusa, through Kepler and Riemann. Hence, in the kind of modified relativistic view associated with Einstein's work, transfiniteness becomes almost merely a method of developing more directly plausible explanations of phenomena situated ontologically within a Cartesian sort of discrete manifold.

This is not an issue merely parallel to the work of the LaRouche-Riemann method in economic science. That difference with Einstein is the crux of the LaRouche-Riemann method.

Those bench-mark observations identified, we focus directly on the fundamental issue of human knowledge. What is man capable of knowing about the lawful composition of his universe, and by what method can such knowledge be adduced with certainty?

Man's existence, first of all, depends entirely on meeting the requirement for increase of per-capita potential relative population-density.

In any fixed technology of productive practice, man's existence is associated with the development and exploitation of a rather well-defined range of natural resources. The development of the raw materials required for human existence on the existing scale, requires some portion of the total available labor of society. If the per-capita such labor required to meet per-capita human needs rises, then the level of human existence must fall. This fall, associated with the depletion of those kinds of natural resources practically accessible to existing levels of technology, means a collapse of the per-capita potential relative population-density. As that potential falls below the existing level of population, the society must collapse.

For example, a hunting-and-gathering mode of human existence can not sustain a population-density of habitable areas much above one person for each 10 to 15 square kilometers, a worldwide human population in the order of about ten millions individuals.

As I have indicated in *The Toynbee Factor*, the take-off point for development of culture above the approximate level of hunting-and-gathering modes of existence begins with fishing in the vicinity of mouths of river-systems. Some of the implications are obvious to anyone who has actually struggled to move about in a raw jungle or forest. The development of boats, and the development of shore settlements near mouths of rivers, is virtually the precondition for development of human culture. This is the best condition for development agriculture in original form, and diffusion up along river-systems into the interior removed from the coast.

Such maritime-riparian developments, of the sort which tend to produce astronomical-navigational science in its earliest rigorous forms, is virtually the indispensable precondition for advancement of the world's human population above the range of a few millions individuals. If a maritime, fishing culture did exist in the Arctic region in such a period as the Ice Age development implies, this would have been the perfect forcing-condition for development of astronomy, and creating a center for diffusion of culture by maritime colonization into other regions of the world, as the account of Diodorus Siculus, of Manetho, and of references to accounts of the temple of Ammon argue.

Once that transition to a culture sustaining above approximately 10 millions potentially occurs, technological progress in the development of the productive powers of labor becomes the precondition for avoiding cultural-population collapse. From this standpoint, the injunction of the Book of Genesis, to "Be fruitful and multiply, and fill the earth and subdue it," is not to be viewed as a whim of the composer of the Book, but as a statement of the most fundamental scientific principle, the precondition for successful continuation of human existence.

The advancement of technology has two effects upon per-capita potential relative population-density. First, the increase in per-capita productivity offsets the effects of depletion of richest, most accessible forms of natural resources in use. Second, more fundamentally, technological progress ultimately, successively, enlarges the range of usable natural resources economically available to society.

From this point of reference, we despise the assertion that human knowledge is typified by perfecting an existing technological mode of repeated, unchanging practice. To the extent that we merely repeat, more rigorously, the same technology, mankind dies. A zero-technological growth policy of guild-like practice, is the practice of a society which lacks the moral fitness to survive. We must despise notions respecting knowledge which bear upon measurement of repeatable actions.

Human knowledge must be situated with regard to means by which improved technology is developed. What we must adduce, are principles proven to guide us to successful revolutions in technology. What we require are principles of hypothesis, principles which are characteristically common to successful, successive scientific revolutions in the productive powers of labor. In other words, we require principles of hypothesis which correspond to the synthetic powers of necessary reason.

Refer now again to the series *P*, cited earlier. Let each level of per-capita potential relative population-density—P_1, P_2, P_3,..., P_i,..., P_m—define a range of technology. Let us associate each such range of technology with a corresponding set of analytical "laws" of the Cartesian form. What we must examine, to adduce the required generalized form of potential-function, is the transformation insets of analytical laws associated with successions of the order P_1, P_2,.... It is the characteristic transformation subsuming such successive P_i's which implies the required potential-function.

If the practice of a society which meets such a generalized requirement, satisfies such a potential-function, we measure resulting success as an increase in per-capita potential relative population-density. The "world-line" associated with such a process of increasing potential signifies people producing an increased population of people of increasing per-capita power to accelerate the rate of increase of such potential. Again, the kind of self-reflexive potential-function as we defined the point earlier. The increase of population according to this requirement, is therefore the fundamental empirical measurement for all human scientific knowledge. The transfinite ordering of scientific progress which subsumes such a potential-function, becomes uniquely the empirical basis for determining what does and what does not represent human scientific knowledge.

This brings us to the crucial point of elementary rigor.

Man demonstrates his knowledge of the lawful composition of the universe to the extent man willfully increases his power over the universe. So, the form and content of scientific knowledge can be nothing else but the kind of practice which directly correlates with that increase of power, with the production of those kinds of discovery, those scientific revolutions, by means of which per-capita potential is increased.

This signifies, additionally, that the degree to which man's knowledge subsumes such successive scientific revolutions, is the expression of the agreement between man's willful practice and the actual ordering of the universe. Those principles of hypothesis—of synthetic necessary reason—which correlate with, and express this connection, this agreement, are the substance of science, and the proper definition of science. Such a notion of science corresponds directly to Plato's notion of the hypothesis of the higher hypothesis, and corresponds in practice to that insight into the Five Platonic Solids from which all modern science's fundamental achievements have sprung over the course of the recent five centuries.

Man must prove in practice that his ideas about the observed universe correspond to the way in which the universe is ordered in fact. One must convert man's judgment of the observed phenomena into the for in of a purposeful, willful action upon the universe. Scientific knowledge must never be degraded merely to a plausible explanation, description of observed phenomena. Any purported "explanation" must be elaborated in the form of a statement which guides mankind to increase its power over the universe. The increase of man's power over the universe is the conditional proof of the "explanation" developed in that active, non-contemplative form.

The only durable measurement of increase of man's power over the universe is increase in per-capita potential relative population-density of society. Since what we must test is not merely individual discoveries, but a "repeatable" method for effecting the kinds of scientific revolutions leading to such increases in per-capita potential, a statement is scientific only to the extent it is a statement of principles of hypothesis of the higher hypothesis.

It is valid statements, so proven, of that higher form, which correlate with the lawful composition of the universe, and which address directly the efficient, ontological reality of the universe.

In this given sense, an economic science based on per-capita potential is the mother of all science, and is the authority to which all other aspects of scientific inquiry must appeal on behalf of their own conditional authority.

The notion of the Logos, the hypothesis of the higher hypothesis is a notion of universal transfiniteness, a notion to the effect that the real universe (the Self-Composer) is ontologically transfinite. It is the continuous manifold, corresponding to that ontological transfiniteness which is the real universe in which the action of cause and effect is immediately situated.

We shall refer to the implications of what we have just developed here at appropriate later locations in this report.

To be continued.[1]

[1] [See "Part II: What Are Economic Shock Waves?" *EIR*, Vol. 9, No. 47, Dec. 7, 1982.]